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DYNAMICS OF NORMAL SEPARATION CRACK GROWTH DURING ITS CLEAVAGE BY A FLOW GAS*

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The propagation of a plane normal separation crack at constant velocity in an elastic medium subjected to a cleaving compressible gas flow is considered. The selfsimilar solution is constructed. The possibility of the formation of stagnation shocks in the gas flow is shown.

Fracture of transparent materials subjected to powerful focused laser radiation occurs, as a rule, with the formation of cracks /1, 2/. Light absorption by transparent materials is accompanied by the formation of high temperature and pressure domains, resulting in the destruction of the material and in gas bubble formation. Cleavage of the material and crack formation occur under the effect of excess gas pressure. The gaseous destruction products penetrate from the cavity into the crack. As time elapses the total pressure on the crack edge increases resulting in further crack growth under the action of the cleaving gas flow. Such a pattern is also realized in the method of intensification of oil and gas extraction during stratum rupture by powder gases /3/, when the gases act on the crack surface similar to a wedge; the cracks being formed here can have fairly large dimensions.

It is established experimentally that under high gas pressure in an underground cavity during a long time interval crack propagation from a cavity occurs at a constant velocity and then decreases slowly /2, 4/.

Selfsimilar problems on crack development at a constant rate have been examined in a number of papers /5-8/. The state of stress of a rectilinear isolated slit growing at a constant rate and loaded from within by concentrated forces is studied in /6/. Solutions are obtained for the plane /7/ and axisymmetric problems /8/ of crack propagation in an elastic medium subjected to a homogeneous tensile stress.

1. Formulation of the problem.

The motion of an isothermal gas penetrating a crack is described by the equations of conservation of mass and momentum

$$\frac{\partial}{\partial t}(\rho w)+\frac{\partial}{\partial x}(\rho w u)=0,\quad P=c^2\rho \tag{1.1}$$
$$\rho\left(\frac{\partial}{\partial t}u+u\frac{\partial}{\partial x}u\right)+\frac{\partial}{\partial x}P=-F$$

(P, ρ are the gas pressure and density, u is the velocity of gas motion in the crack, c is the isothermal velocity of sound, w is the crack aperture, and F is the drag force during gas motion along the crack).

For high Reynolds numbers $(uw/\nu>750)$ the drag force is described by the quadratic law

$$F={}^1/_4\lambda_s w^{-1}\rho u\mid u\mid \tag{1.2}$$

where λ_s is a coefficient which depends on the Reynolds number and the crack roughness in the general case.

The elastic displacements of the medium satisfy the equations /5-6/

$$w_i=u_i+v_i,\quad \Delta u_i=\frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}u_i,\quad \Delta v_i=\frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}v_i;\quad i=1,2 \tag{1.3}$$
$$\frac{\partial}{\partial y}u_1=\frac{\partial}{\partial x}u_2,\quad \frac{\partial}{\partial x}v_1=-\frac{\partial}{\partial y}v_2\quad \left(\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)$$

Here $u_i(x, y, t)$, $v_i(x, y, t)$ are the potential and solenoidal components of the displacement

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vector $\mathbf{w}(x, y, t)$, and c_1, c_2 are the longitudinal and transverse wave velocities $(c_1 > c_2)$.

The components of the stress tensor are related to the displacements by the following expressions (μ is Lamé's constant)

$$\sigma_{xx} = \mu[(c_1/c_2)^2 \operatorname{div}\mathbf{w} - 2\partial w_2/\partial x] \quad (1.4)$$
$$\sigma_{yy} = \mu[(c_1/c_2)^2 \operatorname{div}\mathbf{w} - 2\partial w_1/\partial x], \ \sigma_{xy} = \mu \operatorname{div}\mathbf{w}$$

The initial and boundary conditions for (1.3) and (1.4) that describe the propagation of a slit in an elastic medium have the form (v is the velocity of the crack)

$$\sigma_{yy} = -P(x, t), \ \sigma_{xy} = 0; \ y = 0, \ |x| \leqslant vt \quad (1.5)$$
$$w_2 = 0, \ \sigma_{xy} = 0; \ y = 0, \ vt < |x| < c_1 t$$

We assume that the stress tensor component σ_{yy} at the crack tips has a root singularity

$$\sigma_{yy}(x, y=0, t) \xrightarrow[x \to vt]{} \frac{K_I(t)}{\sqrt{2\pi(x - vt)}} \quad (1.6)$$

where $K_I(t)$ is the stress intensity factor.

We use the following boundary conditions for system (1.1) describing the motion of the gas in a propagating crack:

$$P(x = 0, t) = P_0, \ u(x = 0 \pm 0, t) = \pm u_*, \ u(x = \pm l(t), t) = \pm v \quad (1.7)$$

(P_0, u_* are constants and $2l(t)$ is the crack length at the time t).

To solve system (1.1) for a subsonic gas influx into the crack $(u_* < c)$ at the point $x = 0$, one boundary condition is sufficient, say, the first from (1.7), while two conditions (1.7) are necessary at this point /9/ for a supersonic influx $(u_* > c)$.

2. The selfsimilar problem. Problem (1.1)-(1.7) is selfsimilar with the variable $\xi = x/(vt)$. The dynamic characteristics, the pressure, velocity, and crack aperture, can be represented in the form

$$P(x,t) = P_0 P_a(\xi), \ u(x, t) = v u_a(\xi), \ \varepsilon = (v/c)^2 \quad (2.1)$$
$$w(x,t) = w_0 t w_a(\xi), \ w_0 = {}^1/_4 \lambda_s v \varepsilon$$

where w_0 is the dimensionality constant of the velocity.

Eqs.(1.1) and conditions (1.7), written in the selfsimilar variables (2.1), have the form (henceforth the subscript a on the selfsimilar variables is omitted)

$$d\ln P/d\xi = -\varepsilon(u - \xi) du/d\xi - u^2 w^{-1} \quad (2.2)$$
$$\frac{du}{d\xi} = \frac{(u - \xi) w^{-1} [u^2 - dw/d\xi] - 1}{1 - \varepsilon (u - \xi)^2}$$
$$P(\xi = 0) = 1, \ u(\xi = 0 \pm 0) = \pm u_*/v, \ u(\xi \to \pm 1) = 1 \quad (2.3)$$

The Hugoniot conditions for system (1.1) can be represented in selfsimilar variables in the form

$$[P(u - \xi)] = 0, \ [P + \varepsilon (u - \xi)^2 P] = 0 \quad (2.4)$$
$$([f] = f(\xi + 0) - f(\xi - 0))$$

To determine the crack profile $w(\xi)$ under the loading $\sigma_{yy} = -P_0 P(\xi)$ we use the method of functionally invariant solutions of Smirnov-Sobolev /5, 6/ by introducing the functions

$$U_i = \partial u_i/\partial t, \ V_i = \partial v_i/\partial t \quad (2.5)$$

The function U_i here satisfies the wave equation for the longitudinal waves while V_i satisfies the transverse wave equation. These functions can be represented as the real parts of analytic functions of the complex variables z_1 and z_2

$$U_i\left(\frac{x}{c_1 t}, \frac{y}{c_1 t}\right) = \operatorname{Re} U_i^1(z_1), \quad V_i\left(\frac{x}{c_1 t}, \frac{y}{c_1 t}\right) = \operatorname{Re} V_i^1(z_2)$$
$$\left(z_k = \frac{xt - iy\sqrt{t^2 - c_k^{-2}(x^2 + y^2)}}{x^2 + y^2}, \quad k = 1, 2\right)$$

Satisfying the last two relationships of system (1.3) and the condition $\sigma_{xy} = 0$ for $y = 0$, we express $V_i^1(z)$, $U_i^1(z)$ in terms of the analytic function $W(z)$ (the prime denotes the derivative with respect to z)

$$U_1^{1'} = \frac{z(c_2^{-2} - 2z^2)}{c_2^{-2}\sqrt{c_1^{-2} - z^2}} W', \quad U_2^{1'} = -\frac{2z\sqrt{c_2^{-2} - z^2}}{c_2^{-2}} W' \tag{2.6}$$

$$V_1^{1'} = \frac{c_2^{-2} - 2z^2}{c_2^{-2}} W', \quad V_2^{1'} = \frac{2z}{c_2^{-2}} W'$$

For $y = 0$ we have from (1.4)

$$\frac{\partial}{\partial t}\sigma_{yy} = \frac{\mu}{t}\operatorname{Re}\left[-\frac{S(z)zW'}{c_2^{-2}\sqrt{c_1^{-2} - z^2}}\right] \tag{2.7}$$

$$S(z) = (c_2^{-2} - 2z^2)^2 + 4z^2\sqrt{c_1^{-2} - z^2}\sqrt{c_2^{-2} - z^2}$$

We formulate the boundary value problem for the function W' in the plane of the complex variable z. Taking account of (2.7) we obtain from (1.5)

$$\operatorname{Im} z = 0, \quad |\operatorname{Re} z| > v^{-1}, \quad \operatorname{Im} W' = -\frac{tc_2^{-2}\sqrt{z^2 - c_1^{-2}}}{\mu z S(z)} y\sigma_{yy}/\partial t \tag{2.8}$$

$$\operatorname{Im} z = 0, \ c_2^{-1} < |\operatorname{Re} z| < v^{-1}, \ \operatorname{Re} W' = 0$$

$$\operatorname{Im} z = 0, \ |\operatorname{Re} z| < c_2^{-1}, \ \operatorname{Re} W' = 0$$

Therefore, problem (1.3)-(1.6) reduces to a mixed Keldysh-Sedov problem for the function W' whose solution can be represented in the form /6/

$$W' = \frac{c_1 z}{v^3(v^{-2} - z^2)^{3/2}}\left\{\frac{1}{\pi}\int_{-v/c_1}^{v/c_1} ds \frac{(s^2 - v^2/c_1^2)^{3/2}\operatorname{Im} W'}{s^2(s - 1/(c_1 z))} + i\frac{A}{c_1 z}\right\} \tag{2.9}$$

The constant A is found from the equation

$$\sigma_{yy}(vt - 0, \ 0, \ t) = -P(vt, \ t) \tag{2.10}$$

We approximate the gas pressure in the crack by the finite sum

$$P(x, t) \simeq P_0 \sum_{j=1}^{M} P_j[\delta_0(\xi_j - \xi) - \delta_0(-\xi_j - \xi)] \tag{2.11}$$

where $\delta_0(x)$ is the Heaviside unit step function.

The equations of elasticity theory are linear; consequently, the profile of the crack opening and the stress intensity factor are also approximated by finite sums

$$w(\xi) \simeq \sum_{j=1}^{M} P_j w^j(\xi), \quad K_I(t) \simeq \sum_{j=1}^{M} P_j K_I^j(t) \tag{2.12}$$

Here $w^j(\xi)$, $K_I^j(t)$ are the crack opening and the stress intensity factor that correspond to the load $P_0[\delta_0(\xi_j - \xi) - \delta_0(-\xi_j - \xi)]$. We find $w^l(\xi)$ and $K_I^l(t)$ for the special loading case

$$P(x, \ t) = P_0[\delta_0(\xi_l - \xi) - \delta_0(-\xi_l - \xi)] \tag{2.13}$$

We have from (2.8) for $y = 0$ ($\delta_1(x)$ is the delta-function)

$$\operatorname{Im} W' = \frac{P_0 c_1^2 n^2 m^2 \xi_l^5}{\mu R(\xi_l)}\sqrt{1 - n^2\xi_l^2} \times$$

$$\left\{\delta_1\left(\frac{1}{vz} - \xi_l\right) - \delta_1\left(\frac{1}{vz} + \xi_l\right)\right\}, \quad n = v/c_1, \quad m = v/c_2$$

$$R(\xi_l) = (2 - m^2\xi_l^2)^2 - 4\sqrt{1 - n^2\xi_l^2}\sqrt{1 - m^2\xi_l^2}$$

Substituting this expression into (2.9) and introducing the dimensionless variable $s = vz$, we obtain

$$\frac{dW}{ds} = \frac{Q(\xi_l)s^2}{(s^2 - 1)^{3/2}}\left\{\frac{2\xi_l^{-1}}{s^2 - \xi_l^{-2}} - \frac{K}{s^2}\right\} \tag{2.14}$$

$$Q(\xi_l) = c_1\frac{P_0}{\mu}\frac{nm^2\xi_l^2}{\pi R(\xi_l)}\sqrt{1 - n^2\xi_l^2}(1 - \xi_l^2)^{3/2}$$

Integrating relationship (2.14), we obtain

$$W = a_0 + a_1 \left\{ \frac{Bs}{\sqrt{s^2-1}} + \frac{2}{\xi_l \sqrt{1-\xi_l^2}} \ln \frac{\sqrt{1-\xi_l^2} + \sqrt{1-s^{-2}}}{\sqrt{\xi_l^2 - s^{-2}}} \right\} \tag{2.15}$$
$$a_1 = Q(\xi_l)\,\xi_l^2 (1-\xi_l^2)^{-1}, \quad B = [K(1-\xi_l^2) + 2\xi_l]\,\xi_l^{-2}$$

The stress tensor component σ_{yy} is determined from (2.7) and (2.14) on the crack continuation

$$\sigma_{yy} = -P_0 \frac{\xi_l^2 \sqrt{1-n^2\xi_l^2}\,(1-\xi_l^2)^{3/2}}{\pi R(\xi_l)} \times \tag{2.16}$$
$$\operatorname{Re} \int_n^s d\eta \frac{\eta^6 \operatorname{Re}(\eta^{-1})}{\sqrt{\eta^2 - n^2}\,(1-\eta^2)^{3/2}} \left[\frac{2\xi_l^{-1}}{\eta^2 - \xi_l^{-2}} - \frac{K}{\eta^2} \right]$$

From the boundary conditions for the analytic function

$$s \to \infty, \ \operatorname{Re} W(s) = O(1), \ \operatorname{Im} W = 0 \tag{2.17}$$
$$\operatorname{Im} s = 0, \ |\operatorname{Re} s| > 1, \ \operatorname{Im} W = 0$$
$$s \to \pm 1, \ W(s) = O\left[(s^2-1)^{-1/2}\right]$$

and (2.15) we obtain $a_0 = 0$. Knowing the potential W (2.15) we find the crack profile from (2.5) and (2.17). Since

$$w^l(x,t) = w_3^l(x, y=0, t) = tw_0 w^l(\xi) = \frac{x}{v} \int_1^{\xi^{-1}} ds \operatorname{Re} W(s)\,|_{\operatorname{Im} s = 0}$$

then

$$w^l(\xi) = \sigma_1 \sqrt{1-\xi^2} - \frac{\sigma_2}{\xi_l^2 \sqrt{1-\xi_l^2}} \varkappa(\xi) \tag{2.18}$$
$$\varkappa(\xi) = |\xi| \ln \frac{|1 - \zeta\xi_l / |\xi||}{1 + \zeta\xi_l / |\xi|} - \xi_l \ln \frac{|1-\zeta|}{1+\zeta}$$
$$\zeta = \left(\frac{1-\xi^2}{1-\xi_l^2} \right)^{1/2}, \quad \sigma_1 = \frac{a_1 B}{w_0}, \quad \sigma_2 = \frac{a_1}{w_0}; \quad -1 \leqslant \xi \leqslant 1$$

Using the condition (2.10), we obtain from (2.16)

$$K = \frac{I_1}{I_2}, \quad I_k = \operatorname{Re} \int_n^{1+0} d\eta \frac{R(\eta^{-1})}{\sqrt{\eta^2 - n^2}\,(1-\eta^2)^{3/2}} J_k; \quad k = 1, 2 \tag{2.19}$$
$$J_1 = 2\eta^6 \xi_l^{-1} (\eta^2 - \xi_l^{-2})^{-1}, \quad J_2 = \eta^4$$

The integrals I_1 and I_2 at the point $\eta = 1$ are understood in the principal value sense and are expressed in terms of elliptic integrals (I_2 /6/)

$$I_1 = 2\xi_l^{-1} \{ n^2 [4 - q_1^{-2} (m^2 - 2n^2)^2 (\xi_l^{-2} - n^2)^{-1}] F(q_1) -$$
$$4m^2 F(q_2) + [4 - q_1^{-2} (m^2 - 2)^2 (\xi_l^{-2} - 1)^{-1}] E(q_1) -$$
$$4(1 - 2\xi_l^2)(1 - \xi_l^2)^{-1} E(q_2) + \xi_l^{-1} b_1 (m^2 \xi_l^2 - 2)^2 (1 -$$
$$n^2 \xi_l^2)^{-1} [\Pi(k_1 d_1, d_1) - \Pi(k_1^{-1} d_1, d_1)] -$$
$$4\xi_l^{-1} b_2 [\Pi(k_2 d, d_2) - \Pi(k_2^{-1} d_2, d_2)] \}$$
$$q_k = \sqrt{1 - i_k^2}, \ b_k = 4i(1 + i_k)^{-1} (1 - \xi_l^2)^{-1}$$
$$k_k = (1 + i_k \xi_l)/(1 - i_k \xi_l), \ d_k = (1 - i)/(1 + i_k); \ i_1 = n,$$
$$i_2 = m$$

where $F(q)$, $E(q)$, $\Pi(k, q)$ are the complete elliptic integrals of the first, second, and third kinds, respectively.

The stress intensity factor for the load (2.13) is found from (2.16)

$$K_I^l(t) = P_0 \sqrt{\frac{vt}{\pi}}\, \xi_l^2 (1 - \xi_l^2)^{3/2} \left(\frac{1 - n^2\xi_l^2}{1 - n^2} \right)^{1/2} \frac{R(1)}{R(\xi_l)} \times \tag{2.20}$$
$$\left[\frac{2\xi_l}{\xi_l^2 - 1} - \frac{I_1}{I_2} \right]$$

For $\xi_l = 1$ the solution of the problem of uniformly loaded crack propagation

$$w(x, t) = -tP_0 c_1 \mu^{-1} n m^2 I_2^{-1} \sqrt{1 - \xi^2}$$

follows from (2.18), (2.19), (2.15), and (2.20).

Problem (2.2), (2.3) of the propagation of a crack wedged open by a gas flow was solved by a fourth-order Runge-Kutta method.

At the beginning of the computation the crack profile was given in the form $w^{(0)} = 1 - \xi$ $(0 \leqslant \xi \leqslant 1)$. The gas pressure and the velocity of its motion in the crack were determined by solving system (2.2) with condition (2.3). For the subsonic gas flow regime in the crack one of the boundary conditions (2.3) was used. In this case the solution of system (2.2) was found with a reverse step from $\xi = 1$ to $\xi = 0$. To solve (2.2) for the supersonic gas flow regime in the crack both boundary conditions (2.3) were taken into account. The integration of (2.2) was here carried out from the two ends of the interval $[0, 1]$. The solutions were merged in the case when the Hugoniot condition (2.4) was satisfied. According to the computed gas pressure in the crack the values of $P_l (l = 1, \ldots, N$, where N is the number of mesh nodes in $\xi)$ were found. The crack aperture $w^{(1)}$ and the stress intensity factor $K_I^{(1)}$ were restored by means of (2.12).

The values of $w_j^l = w^l(x_j)$ and K_I^l were calculated at the beginning of the computation $(l, j = 1, \ldots, N)$. Then the algorithm for the solution was repeated with a new profile $w^{(1)}$ etc., until the following condition was satisfied: $\| w^{(s+1)} - w^{(s)} \| \leqslant \delta$ $(\delta = 10^{-5}$, s is the number of iterations). Convergence was achieved after 6-7 iterations.

3. Results and their discussion. Let us examine the results of the numerical solution of problem (2.2), (2.3), (2.11), (2.12), (2.18), (2.20).

When the crack propagation velocity in an elastic medium v is greater than the isothermal speed of sound in the cleaving gas flow, only a supersonic regime of gas influx into the crack is possible $(u_* > c)$.

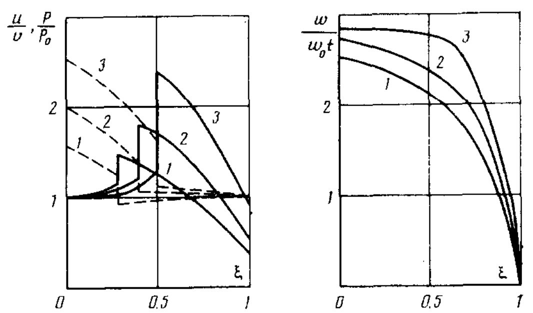

Fig.1

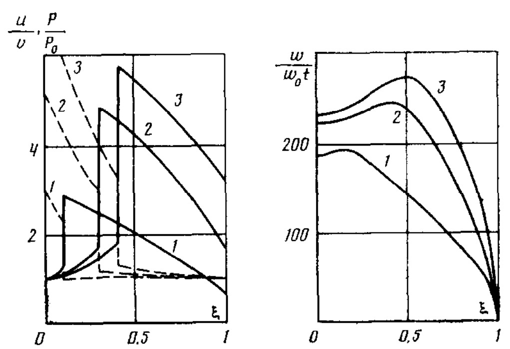

Fig.2

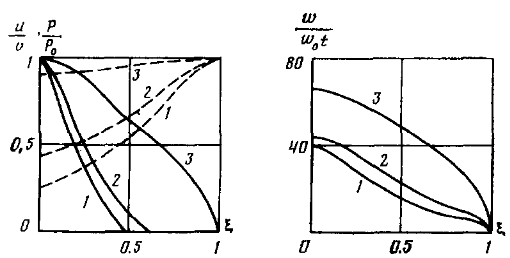

Fig.3

Fig.1 illustrates the profile of the crack opening $w/(w_0 t)$, the pressure P/P_0 (solid curves) and the gas flow velocity u/v (dashed lines) for $u_*/c = 1.6, 2.0, 2.5$ (curves *1-3*). These calculations were carried out for the following values of the parameters: $n = 0.2$, $m = 0.3$, $\varepsilon = 1.5$, $P_0/\mu = 0.33$. It is seen that the gas velocity in the crack decreases as ξ increases, while the pressure first increases and then decreases. For a certain value $\xi = \xi_0$ both the pressure and the velocity vary abruptly, after which the velocity of motion of the gas is practically equal to the velocity of motion of the crack $(u \simeq v)$.

The jump in the solution when $\xi = \xi_0$ is a stagnation shock (SS) whose front is opposite to the gas flow. The origination of the SS is explained by the action of the drag force on the supersonic gas flow /9, 10/. The decrease in the velocity of gas influx into the crack results in a decrease in the magnitude of the jump on the SS. For a crack with $v > c$ there exist $u_{\min}$ and $\xi_{\min}$ such that for $u_* > u_{\min}$ the SS location corresponds to $\xi_0 > \xi_{\min}$, while the SS is degenerate for $u_* = u_{\min}$ and the gas pressure and velocity profiles in the crack become continuous. Solutions with $u_* < u_{\min}$ do not exist.

If the crack velocity of propagation v is less than the isothermal speed of sound c, two gas influx regimes are possible in the crack: supersonic $(u_* > c)$ and subsonic $(u_* < c)$.

For $u_* > c$ there is always a SS in the flow (Fig.2 is constructed for $m = 0.173$; $n = 0.115$; $\varepsilon = 0.5$; $P_0/\mu = 2.3 \cdot 10^3$, $u_*/c = 2.98$ (curve *1*); $P_0/\mu = 1.95 \cdot 10^{-3}$, $u_*/c = 5.36$ (curve *2*); $P_0/\mu = 1.87 \cdot 10^{-3}$, $u_*/c = 7.05$ (curve *3*)). As the gas influx velocity decreases $u_* \to u_{\min}$, the SS front $\xi_0 \to 0$.

One of the boundary conditions (2.3) is utilized for the subsonic mode of gas motion in the crack for the solution of (2.2), (2.3) at $\xi = 0$. The velocity of gas influx into the crack is determined as a result of solving the problem. The gas pressure and velocity profiles are continuous functions of the self-similar variable ξ (Fig.3 is constructed for $m = 0.173; n = 0.115;$ $\varepsilon = 0.5;$ $P_0/\mu = 2.12\cdot10^{-1}$ (curve *1*); $P_0/\mu = 9.17\cdot10^{-2}$ (curve *2*); $P_0/\mu = 6.07\cdot10^{-3}$ (curve *3*)).

If the crack rate of development is much less than the speed of sound in an elastic medium $(m^2, n^2 \ll 1)$, then keeping the principal term in the expansion (2.18) in the small parameters m^2 and n^2 we obtain

$$w^l(\xi) = \frac{P_0}{\pi\mu}\frac{v}{w_0}\frac{(\lambda+2\mu)}{(\lambda+\mu)}\{(\arcsin\xi_l)\sqrt{1-\xi^2}+\varkappa(\xi)\} \tag{3.1}$$

where λ, μ are Lamé coefficients, $c_1 = [(\lambda + 2\mu)/\rho_s]^{1/2}$; $c_2 = [\mu/\rho_s]^{1/2}$ is the speed of sound in the medium, and ρ_s is the rock density.

Relationship (3.1) governs the crack opening profile $w^l(\xi)$ for the load (2.13) in the quasistationary approximation.

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